

THE BALMER LINES OF HYDROGEN

Last Revision: August 7, 2007

QUESTION TO BE INVESTIGATED:

What is the value of the Rydberg constant for hydrogen?

INTRODUCTION:

In this experiment a diffraction grating spectroscope and two different light sources will be used to make a high precision measurement of the Rydberg constant for Hydrogen. You will also be introduced to fitting using linear least squares minimization techniques.

THEORY:

The Balmer series in hydrogen is the set of lines produced when an excited hydrogen atom decays to the $n=2$ level from a higher n state. The wavelength λ of emitted light is given by the formula:

$$\frac{1}{\lambda} = R_h \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1)$$

where R_h is the Rydberg constant and n_i and n_f are the quantum numbers of the initial and final states of the hydrogen atom. In our case, where $n_f=2$, one can easily see the first 3 lines in a spectroscope. If the wavelength λ is measured and n is appropriately chosen, then the Rydberg constant may be determined for the given decay.

The above formula was derived from the Bohr treatment of the atom and is accurate to about 1 part in 10^5 , which is well matched with our current apparatus.

Light may be separated into its constituent wavelengths by a diffraction grating. The grating used in this experiment is reflective with approximately 600 grooves/mm. Visible light incident on the grating is diffracted into several different orders. The 0th order is just specular reflection. There is no separation of wavelength in this order, so the first order is the first useful diffracted order. The grating equation is,

$$m\lambda = a(\sin\theta_i + \sin\theta_m) \quad (2)$$

where m is the order number ($m=0,1,-1,2,-2,\dots$), λ is the wavelength, a is the groove spacing on the grating, θ_i is the angle of incidence, and θ_m is the angle of the m^{th} diffracted order. The following diagram shows the diffraction of the $m=1,0,-1$ orders. Higher orders may also be visible but their presence depends on the number of grooves illuminated and on the density of the grooves.

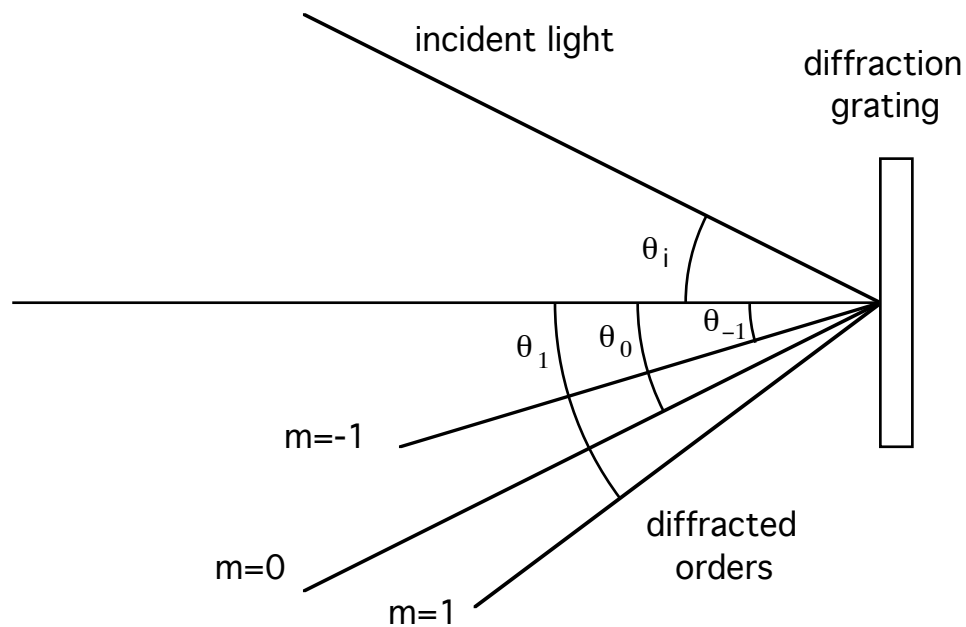


Figure 1. The diffracted paths of light.

PROCEDURE:

You will use a mercury and a hydrogen light source. The mercury source will help you calibrate your device so that you can measure the Rydberg constant of hydrogen. There are six prominent lines in mercury ranging from 4047 to 5791 angstroms. If you're lucky you may also see three red lines too. These are very faint and can significantly improve your calibration if they are visible to you. Otherwise, use the purple lines of the next order to the far left of the first order lines. Place the mercury source as close to the entrance slits as possible. The narrower the slits the better will be the resolution (sharpness) of the lines. However, the lines will become much dimmer as the slits are closed. A balance between these two considerations must be obtained.

Locate the first order series of lines as shown in the figure. These should be the brightest. Swing the grating arm around and roughly center the cross hair in the middle of the distribution. Adjust the arm until it reads zero while staying centered on the green mercury line.

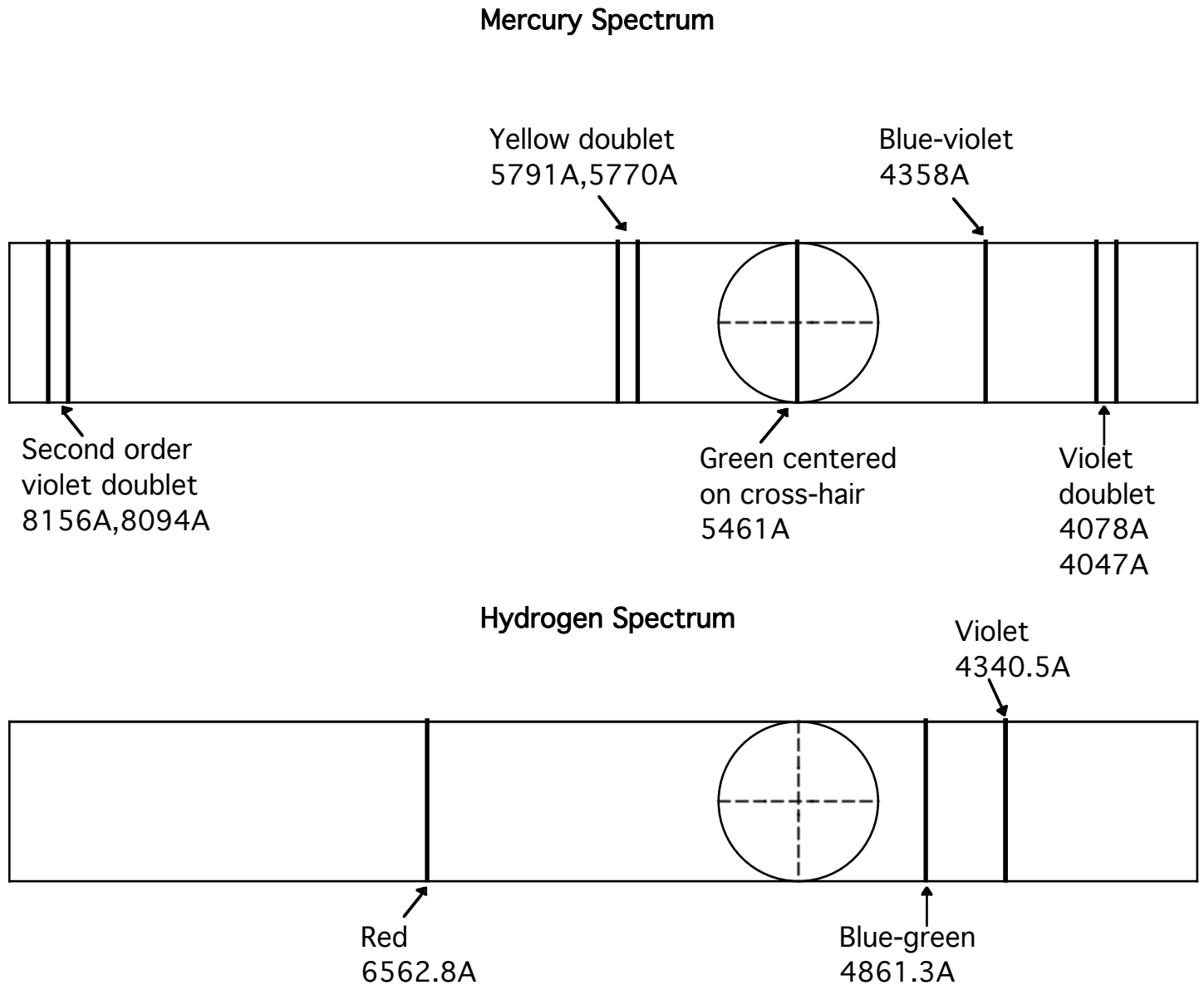


Figure 2. Top: The relative positions of the diffracted mercury lines. Bottom: The relative positions of the diffracted hydrogen spectrum.

Note that the scale goes from minus fifteen to fifteen divisions. Swing the arm to the far left and measure the position of each line while moving from left to right. Do not move backwards. If you bump the apparatus, swing the arm to the far left again and return to the line of interest. Measure each line and estimate as many decimals as is possible. After one reading of each line is made, move the apparatus around,

including the source, grating, and slits. This will randomize any errors due to these items. Repeat the above procedure until you have ten readings of each line.

Now use the hydrogen lamp. Make sure to use the lamp only when taking data, as it has a finite life-time. There should be three lines clearly visible. Use the same procedure as with the mercury source, swinging from left to right. Move only the telescope. Repeat until you have ten readings of each line.

ANALYSIS: Least Squares Tutorial

- Use a spreadsheet to histogram the ten readings for each line that you have measured (10 histograms). Fit this histogram to a Gaussian to determine the pertinent statistical information.
- Create a calibration plot of wavelength vs. angular position using your mercury data. Fit a function to this line. Make sure to justify your fit using a chi squared test.
- Use your fit to extrapolate the wavelengths of your observed hydrogen lines and their errors.
- Calculate the approximate error in the wavelength due to the error in your hydrogen position by multiplying the position error by the slope, evaluated at the proper position. How much error is introduced by ignoring the higher order terms?
- You now have two errors for the hydrogen wavelength, one from the calibration fit and one from the hydrogen position measurement. Since these errors are nearly independent, they may be combined in quadrature as shown below.

$$\delta\lambda = \sqrt{\delta\lambda_{\text{fit}}^2 + \delta\lambda_{\text{hydrogen}}^2}$$

- Using the calculated wavelengths and the total errors, calculate the value of the Rydberg constant and its error using the Eq. (1) for each line.
- Make a weighted average of the 3 values and obtain the final R/h , its error, and reduced chi-square.
- Compare your results with the accepted value. What is the probability that your results are in agreement with the accepted value? Be quantitative.

REFERENCES:

1. Melissinos, A. C., *Experiments in Modern Physics*, pages 28–37.
2. Born, M., *Atomic Physics*, Chapter 5.